INTERMEDIATE SCALE DEPENDENCE OF NON-UNIVERSAL GAUGINO MASSES IN SUPERSYMMETRIC SO(10)

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Abstract

We calculate the dependence on intermediate scale of the gaugino mass ratios upon breaking of $SO(10)$ into the SM via an intermediate group H. We see that the ratios change significantly when the intermediate scale is low (say, 10^8 GeV or 1 Tev) compared to the case when the two breakings occur at the same scale.

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1 Introduction

With the Large Hadron Collider (LHC) having started to operate, the high energy community is expanding focus in the study and search of new physics beyond the standard model (SM).

Grand unification theories (GUTs) are models of the most promising ones for this new physics [1]. However, supersymmetry (SUSY) is necessary here to make the huge hierarchy between the GUT scale and the electroweak scale stable under radiative corrections [2]. In this regard, SUSY SO(10) is an appealing candidate for realistic GUTs [3]. Universal boundary conditions for gaugino masses, as well as other soft terms, at the high scale (the unification scale or Plank scale) are adopted in the the setting of the minimal supergravity (mSUGRA) or the constrained minimal supersymmetric standard model CMSSM [4]. If the discrepancy between the SM theoretical predictions and the experimental determinations of $(g-2)$ is confirmed at the 3-sigma level, this could be interpreted as strong evidence against the CMSSM [5]. Non-universal gaugino masses may arise in supergravity models in which a non-minimal gauge field kinetic term is induced by the SUSY-breaking vacuum expectation value (vev) of a chiral superfield that is charged under the GUT group G [6]. The non-universal gaugino masses

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resulting from SUSY-breaking vevs of non-singlet chiral superfields, for $G = SU(5)$, $SO(10)$ and E_6, and their phenomenological implications have been investigated in [7, 8, 9, 10].

If the grand unification group G is large enough, like $SO(10)$ or E_6, then there are more than one breaking chain from G down to the SM. It is natural here to assume that there exist multi intermediate mass scales in the breaking chain. It has been found that when extrapolating the coupling strengths to very high energies, they tend to converge in the non-SUSY $SO(10)$ provided one introduces two new intermediate energy scales, whereas they do not meet at one point in the absence of intermediate energy scale [11]. A systematic study of the constraints of gauge unification on intermediate mass scales in non-SUSY $SO(10)$ scenarios was recently discussed in [12].

The possibility of the existence of intermediate scales is an important issue for supersymmetric unification. The success of the minimal supersymmetric standard model MSSM couplings unification [13] favors a single GUT scale, and the intermediate scales cannot be too far from the GUT scale. However, recent studies show that in GUTs with large number of fields renormalization effects significantly modify the scale at which quantum gravity becomes strong and this in turn can modify the boundary conditions for coupling unification if higher dimensional operators induced by gravity are taken into consideration [14]. In GUT model building, the so called magic fields can be used to fix the gauge coupling unification in certain two-step breakings of the unified group [15]. It has been pointed out that any choice of three options - threshold corrections due to the mass spectrum near the unification scale, gravity induced non-renormalizable operators near the Plank scale, or presence of additional light Higgs multiplets - can permit unification with a lower intermediate scale [16]. This unification with distinct energy scales yields right handed neutrino masses in the range ($10^8 - 10^{13}$ GeV) relevant for leptogenesis [17], perhaps even reaching the TeV region [16].

In the previous studies [7, 8, 9, 10] on non-universal gaugino masses in SUSY-$SO(10)$ one assumed for simplicity that there was no intermediate scales between M_{GUT} and M_S (the SUSY scale$\sim 1TeV$) or the electro-weak scale M_{EW}. In this paper, we study in detail the intermediate scale dependence of the non-universal gaugino masses.

The starting point is to consider a chiral superfield ('Higgs' field) Φ transforming under the gauge group $G = SO(10)$ in an irrep R lying in the symmetric product of two adjoints *:

$$(\mathbf{45} \times \mathbf{45})_{symmetric} \quad = \quad \mathbf{1} + \mathbf{54} + \mathbf{210} + \mathbf{770} \tag{1}$$

If R is G non-singlet and Φ takes a vev (vacuum expectation value) spontaneously breaking G into a subgroup H containing the SM, then it can produce a gauge non-singlet contribution to the H-gaugino mass matrix [19]

$$M_{\alpha,\beta} \quad = \quad \eta_\alpha \delta_{\alpha\beta} \langle \Phi \rangle \tag{2}$$

where the discrete η_α's are determined by R and H.

Here, we make two basic assumptions. The first one is to omit the 'possible' situation of a linear combination of the above irreps and to consider the dominant contribution to the gaugino masses coming from one of the non-singlet F-components. The second assumption is that $SO(10)$ gauge symmetry group is broken down at GUT scale M_{GUT} into an intermediate group H which, in turn, breaks down to the SM at some intermediate scale M_{HB}. In the case of several intermediate symmetry breakings one can assume various intermediate scales, for which case it is straightforward to generalize our method.

We insist on H being the gauge symmetry group in the range from M_{HB} to M_{GUT}. Thus, only the F-component of the field Φ which is neutral with respect to H can acquire a vev yielding gaugino masses. Depending on the breaking chain one follows down to the SM, ratios of gaugino masses M_a's are dependent of M_{HB} and are determined purely by group theoretical factors only if $M_{HB} = M_{GUT}$.

*All group theory considerations can be found in the reviw article [18].

In fact, the functional dependence on M_{HB} of the gaugino mass ratios can not be deduced from their values obtained in the case of $M_{HB} = M_{GUT}$ by mere renormalization group (RG) running, and one has to consider carefully the normalization of the group generators and the mixing of the abelian $U(1)$'s necessary to get the dependence of the $U(1)_Y$'s gaugino mass on the intermediate scale.

Whereas in ref.[7] we considered only low dimensional irreps **54**, **210**, we extend here our analysis to include all three non-singlet irreps. Moreover, there were some errors in the results of ref.[7], which upon being corrected agree now with the conclusions of [8, 9, 10] when $M_{HB} = M_{GUT}$.

The plan of this paper is as follows. In section 2, we consider the first step of breaking, from $GUT = SO(10)$ to the intermediate group H, and calculate the H-gaugino mass ratios at the GUT scale M_{GUT}, for the three cases $H = G_{422} \equiv SU(4)_C \times SU(2)_L \times SU(2)_R, H = SO(3) \times SO(7)$ and $H = H_{51} \equiv SU(5) \times U(1)_X$, depending on the specific irreps in Eq.(1). We investigate the second step of the breaking, from the intermediate group H to the SM group in section 3, and compute the MSSM gaugino masses in terms of the H-gaugino masses at the intermediate breaking scale M_{HB}. Taking the RG running from M_{GUT} to M_{HB} into consideration, we compute in section 4 the MSSM gaugino mass ratios at M_{HB}. We also state in this section the particle content of the model in each case, and calculate the beta function coefficients necessary for the RG running. In section 5, we summarize the results in form of a table, where we compare numerically the case of two breaking scales with the case of one breaking scale, and present our conclusions.

2 From $GUT = SO(10)$ to the intermediate group H

Here we discuss the different ways in which one can break the GUT-group $SO(10)$ depending on the Higgs irrep one uses. As noted earlier, three irreps can be used (see Eq.1): **54**, **210** and **770**.

2.1 The irrep 54

If an irrep **54** is used then the branching rules for $SO(10)$ tell us it can be broken into several subgroups (e.g. $H = G_{422}, H = SU(2) \times SO(7), H = SO(9)$). The choice $H = SO(9)$ leads to universal gaugino masses whereas the other two possible chains are more interesting.

2.1.1 $H = G_{422}$

The **54** irrep can be represented as a traceless and symmetric 10×10 matrix which takes the vev:

$$< \mathbf{54} > \quad = \quad v\ Diag(\underbrace{2, \ldots, 2}_{6}, \underbrace{-3, \ldots, -3}_{4}) \tag{3}$$

with the indices $1, \ldots 6$ corresponding to $SO(6) \simeq SU(4)_C$ while those of $7, \ldots 0$ (henceforth 0 means 10) correspond to $SO(4) \simeq SU(2)_L \times SU(2)_R$.

This implies that at M_{GUT}-scale we have:

$$\left.\frac{M_L}{M_4}\right|_{M_{GUT}} = \left.\frac{M_R}{M_4}\right|_{M_{GUT}} \quad = \quad -\frac{3}{2} \tag{4}$$

2.1.2 $H = SU(2)_L \times SO(7)$

The first breaking is achieved by giving a vev to the irrep **54**

$$< \mathbf{54} > \quad = \quad vDiag(\underbrace{7/3, \ldots, 7/3}_{3}, \underbrace{-1, \ldots, -1}_{7}) \tag{5}$$

where the indices 1,2,3 correspond to $SO(3) \simeq SU(2)_L$ and $4, \ldots 0$ correspond to SO(7). This gives at M_{GUT}-scale

$$\left.\frac{M_L}{M_7}\right|_{M_{GUT}} = -\frac{7}{3} \tag{6}$$

2.2 The irrep 210

This irrep can be represented by a 4^{th}-rank totally antisymmetric tensor Δ_{abcd}. It can break SO(10) in different ways, of which we consider two.

2.2.1 $H = G_{422}$

The first breaking from $SO(10)$ to H is achieved when the only non-zero vev is $< \Delta_{abcd} >= v\epsilon_{7890} = v$ [20] where $(a, b, c, d \in \{1, \ldots, 0\})$. This leads to the mass term:

$$\mathcal{L}_{\text{mass}} \propto < \Delta_{abcd} > \lambda^a_b \lambda^c_d = \frac{v}{4}[(\lambda^7_8 + \lambda^9_0)^2 - (\lambda^7_8 - \lambda^9_0)^2] \tag{7}$$

As the indices $(1, \ldots, 6)$ which correspond to $SO(6)$ do not appear in the mass term then we have

$$M_4|_{M_{GUT}} = 0 \tag{8}$$

We can take the gauginos $\lambda_{2L}, \lambda_{2R}$ corresponding to $SU(2)_L, SU(2)_R$ as being proportioanl to the 'bracketed' combinations of λ^7_8 and λ^9_0 in Eq.(7), and thus we get:

$$\left.\frac{M_R}{M_L}\right|_{M_{GUT}} = -1. \tag{9}$$

2.2.2 $H = H_{51}$

This breaking from $SO(10)$ occurs when [21]:

$$\begin{aligned}\Delta_{1234} &= \Delta_{1256} = \Delta_{1278} = \Delta_{1290} = \Delta_{3456} = \Delta_{3478} \\ &= \Delta_{3490} = \Delta_{5678} = \Delta_{5690} = \Delta_{7890} = v.\end{aligned} \tag{10}$$

For the $H = H_{51}$-case, we adopt the convention of restricting the use of indices to the $SU(5)$-indices in order to express only the $SU(5) \times U(1)_X$ gauginos amongst the $SO(10)$-ones. In fact, the branching rule

$$\mathbf{10} \overset{SO(10)\supset SU(5)\times U(1)_X}{=} (\mathbf{5})_2 + (\overline{\mathbf{5}})_{-2} \tag{11}$$

allows us to use the indices:

$$i = \tilde{a} + \tilde{b} \equiv a + \bar{b} \quad \text{with} \quad i =\in \{1, \ldots, 0\}; \tilde{a} \equiv 2a - 1 \in \{1, 3, 5, 7, 9\}; \tilde{b} \equiv 2\bar{b} \in \{2, 4, 6, 8, 0\} \tag{12}$$

and so we have the $SU(5)$-indices $(a = 1, \ldots, 5; \bar{b} = \bar{1}, \ldots, \bar{5})$ written usually as an upper index for 'a' and a lower index for 'b' (omitting the 'bar' of $\bar{b}$). With this, we write the $SO(10)$ adjoint irrep $\lambda^{(2a-1)(2b)}$ as λ^a_b using the H_{51} indices $(a, b = 1, \ldots, 5)$.

We know that the only way to get a 4^{th}-rank totally antisymmetric tensor invariant under $SU(5)$ is by considering:

$$\epsilon^{abefg}\epsilon_{cdefg} = \delta^a_c\delta^b_d - \delta^a_d\delta^b_c \tag{13}$$

$(a, b, c, d, e, f, g = 1, \ldots, 5)$ and thus the H_{51}-singlet takes on the invariant form

$$< \Delta^{ab}_{cd} > = v\epsilon^{abefg}\epsilon_{cdefg} \tag{14}$$

The gaugino mass term becomes

$$< \Delta^{ab}_{cd} > \lambda^c_a \lambda^d_b \quad \propto \quad -\widehat{\lambda^c_a}\widehat{\lambda^a_c} + \frac{4}{5}(\lambda^b_b)^2 = -(\widehat{\lambda^c_a})^2 + 4(\lambda)^2 \tag{15}$$

where the 'traceless' $SU(5)$-gaugino $\widehat{\lambda^c_a}$ and the $U(1)_X$-gaugino λ are defined as usual by:

$$\widehat{\lambda^a_b} \quad = \quad \lambda^a_b - \frac{1}{5}\delta^a_b \lambda^c_c \tag{16}$$

$$\lambda \quad = \quad \frac{1}{\sqrt{5}}\lambda^c_c \tag{17}$$

We get at M_{GUT} the ratio:

$$\left.\frac{M_X}{M_5}\right|_{M_{GUT}} \quad = \quad -4 \tag{18}$$

2.3 The irrep 770

This irrep can be represented by a traceless 4$^{\text{th}}$-rank tensor $\phi^{ij,kl}$ with symmetrized and anti-symmetrized indices in the combinations corresponding to the Young diagram with two rows and columns. It can break $SO(10)$ in three ways.

2.3.1 $H = G_{422}$

Here, since we have the branching rule:

$$\mathbf{10} \quad \overset{SO(10)\supset SO(6)\times SO(4)}{=} \quad (\mathbf{1},\mathbf{4}) + (\mathbf{6},\mathbf{1}) \quad \overset{SO(10)\supset SU(4)\times SU(2)\times SU(2)}{\equiv} \quad (\mathbf{1},\mathbf{2},\mathbf{2}) + (\mathbf{6},\mathbf{1},\mathbf{1}), \tag{19}$$

we can set $\phi^a = \phi^\alpha + \phi^i$ with $a = 1, 2, ..., 0;\ \alpha = 1, ..., 6;\ i = 7, ..., 0$. When the scalar components of $\phi^{ab,cd}$, corresponding to the singlet $(\mathbf{1},\mathbf{1})$ of $\mathbf{770}$ under $SO(10) \supset SO(6) \times SO(4)$, acquire a non-zero vev, then the tensor structure impose the form:

$$\begin{aligned} < \phi^{\alpha\beta,\gamma\delta} > &= v(\delta^{\alpha\beta}\delta^{\gamma\delta} - \delta^{\alpha\delta}\delta^{\beta\gamma}) \\ < \phi^{ij,kl} > &= sv(\delta^{ij}\delta^{kl} - \delta^{il}\delta^{jk}) \\ < \phi^{\alpha\beta,ij} > &= s'v\delta^{\alpha\beta}\delta^{ij} \end{aligned} \tag{20}$$

$(\alpha, \beta, \gamma, \delta = 1, \ldots, 6; i, j, k, l = 7, \ldots, 0)$. Forcing the tensors $\phi^{aa\gamma\delta}$ and ϕ^{aaij} to be traceless would imply $s' = -\frac{5}{4}$ and $s = \frac{5}{2}$, and so one gets a mass term:

$$\mathcal{L}_{\text{mass}} \quad = \quad \phi^{\alpha\beta\gamma\delta}\lambda_{\alpha\beta}\lambda_{\gamma\delta} + \phi^{ijkl}\lambda_{ij}\lambda_{kl} \tag{21}$$

$$= \quad -v(\lambda_{\alpha\beta})^2 - sv(\lambda_{ij})^2 \tag{22}$$

The $\lambda_{\alpha\beta}$'s correspond to $SO(6)$-gauginos whereas λ_{ij}'s correspond to $SO(4)$-gauginos, whence we get at M_{GUT}-scale the ratios:

$$\left.\frac{M_L}{M_R}\right|_{M_{GUT}} = 1 \quad , \quad \left.\frac{M_R}{M_4}\right|_{M_{GUT}} = \frac{5}{2} \tag{23}$$

2.3.2 $H = SO(3) \times SO(7) \simeq SU(2)_L \times SO(7)$

Again, the branching rule:

$$\mathbf{10} \quad \overset{SO(10)\supset H_{51}}{=} \quad (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{7}) \tag{24}$$

enables us to set $\phi^a = \phi^\alpha + \phi^i$ with $a = 1, \ldots, 0$; $\alpha = 1, \ldots, 7$; $i = 8, 9, 0$. In the same way as in the case of $H = G_{422}$, when the scalar components of $\phi^{ab,cd}$, corresponding to the singlet $(\mathbf{1}, \mathbf{1})$ of $\mathbf{770}$ under $SO(10) \supset SO(3) \times SO(7)$, acquire a non-zero vev then we have the same tensor structures as in Eqs.(20). Forcing the traces $\phi^{aa\gamma\delta}$ and ϕ^{aaij} to vanish would imply $s' = -2$ and $s = 7$. Substituting in the Lagrangian gaugino mass term gives now at M_{GUT} the ratios:

$$\left.\frac{M_L}{M_R}\right|_{M_{GUT}} = 1 \quad , \quad \left.\frac{M_R}{M_4}\right|_{M_{GUT}} = 7 \tag{25}$$

2.3.3 $H = H_{51}$

Again, using the branching rule in Eq.(11), we can take $\phi^a = \phi^i + \phi^{\bar{k}} \equiv \phi^j + \phi_l$ with $a = 1, \ldots, 0; i = 1, 3, 5, 7, 9 \equiv 2j - 1; \bar{k} = 2, 4, 6, 8, 0 \equiv 2l$ $(j, l = 1, \ldots, 5$ are the $\mathbf{5}$ and $\bar{\mathbf{5}}$ indices respectively). When the traceless 4^{th}-rank tensor $\phi^{ab,cd}$ scalar fields, corresponding to the singlet $(\mathbf{1}, \mathbf{1})$ of $\mathbf{770}$ under $SO(10) \supset H_{51}$, have a non-zero vev, then we have the following tensor structures:

$$\begin{aligned} \phi^{ab,cd} &= \phi^{ij,kl} + \phi_l^{ij,k} + \phi_{kl}^{ij} + \phi_{ij}^{kl} + \phi_{j,kl}^{i} + \phi_{ij,kl} && (26) \\ < \phi^{ij,kl} > &= v_1(\delta^{ij}\delta^{kl} - \delta^{kj}\delta^{il}) \\ < \phi_l^{ij,k} > &= v_2\delta^{ij}\delta_l^k \\ < \phi_{kl}^{ij} > &= v_3(\delta^{ij}\delta_{kl} + \delta_k^i\delta_l^j + \delta_l^i\delta_k^j) && (27) \end{aligned}$$

$(a, b, c, d = 1, \ldots, 0; i, j, k, l = 1, \ldots, 5)$. Note that since $SU(5)$ is the only maximal non-abelian subgroup in H_{51} then all the vevs above are equal $v_1 = v_2 = v_3 = v$. We note also that the contribution to the gaugino mass from the last three terms in Eq.(26) is equal to that coming from the first three terms, and thus we can limit the computation to these latter terms to get the mass term:

$$\langle\phi^{ab,cd}\rangle\lambda_{ab}\lambda_{cd} = v[\widehat{\lambda_l^j}\widehat{\lambda_j^l} + 16\lambda^2] \tag{28}$$

where the expressions of the 'traceless' $SU(5)$-gaugino $\widehat{\lambda_l^j}$ and the $U(1)_X$-gaugino λ are taken from Eqs.(16) and (17). We get at M_{GUT} the ratio:

$$\left.\frac{M_X}{M_5}\right|_{M_{GUT}} = 16 \tag{29}$$

3 From the intermediate group to the SM

We discuss here the second stage of the breaking from H into the SM. We note that in some cases there are more than one $U(1)$-group, and we need to consider the mixing of these $U(1)$'s in order to get the $U(1)_Y$ of the SM. The method is standard and we work it out case by case.

3.1 $H = G_{422} \equiv SU(4)_C \times SU(2)_L \times SU(2)_R \rightarrow SM \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$

The Higgs field responsible for the breaking $SU(4)_C \times SU(2)_R \rightarrow SU(3)_C \times U(1)_Y$ can be taken to include the irrep $(\mathbf{4}, \mathbf{2})$ of the group $SU(4)_C \times SU(2)_R$:

$$\Phi = \varphi^a \bigotimes \varphi^r : a \in \{1, 2, 3, 4\}, r \in \{1, 2\} \tag{30}$$

We can choose Φ to be in the spinor irrep of $SO(10)$ since we have the branching rule:

$$\mathbf{16} \overset{SO(10)\supset G_{422}}{=} (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \tag{31}$$

and we can write the covariant derivative terms related to the $SU(4)_C \times SU(2)_R$ group in the form:

$$\mathcal{D}_\mu \Phi = \partial_\mu \Phi - i g_4 \frac{T^b}{2} A^b \varphi^a - i g_R \frac{\tau^s}{2} B^s \varphi^r \tag{32}$$

where $T^b (b \in \{1, \ldots, 15\})$ are the 4×4 generalized Gellman matrices for $SU(4)$ with the standard normalization $Tr(\frac{T^a}{2}\frac{T^b}{2}) = \frac{1}{2}\delta^{ab}$, $\tau^r (r \in \{1,2,3\})$ are the 2×2 Pauli matrices satisfying $Tr(\frac{\tau^r}{2}\frac{\tau^s}{2}) = \frac{1}{2}\delta^{rs}$.

In order to break $SU(4)_C$ to $SU(3)_C \times U(1)$, and $SU(2)_R$ to $U(1)'$, the Higgs fields take the vevs:

$$\langle \varphi^a \rangle = v_1 \delta^{a4} \quad , \quad \langle \varphi^r \rangle = v_2 \delta^{r1} \tag{33}$$

Since both φ^a and φ^r originate from the same Φ, the spinor irrep in $SO(10)$ which under $SO(10) \supset SM$ has the component $(\mathbf{1}, \mathbf{1})_0$, then the two vevs are equal: $v_1 = v_2 = v$. Concentrating on the mixing of the $U(1)$ from $SU(4)_C$ and the other $U(1)'$ from $SU(2)_R$, we note that the corresponding A^{15} and B^3 components will mix together, and thus we obtain the neutral gauge boson mass terms in the form :

$$\langle D_\mu \Phi \rangle \langle D_\mu \Phi \rangle^+ = \frac{v^2}{4} \left(\sqrt{\frac{3}{2}} g_4 A^{15} - g_R B^3 \right)^2 \tag{34}$$

This quadratic form in the fields B^3 and A^{15} has a zero eigenvalue whose corresponding eigenstate can be identified as the massless $U(1)_Y$ gauge boson E. By diagonalizing the corresponding mass matrix we obtain the two physical vector bosons: the massless gauge boson E , and the orthogonal combination F corresponding to a massive vector boson:

$$\begin{aligned} F &= \cos\theta A^{15} - \sin\theta B^3 \\ E &= \sin\theta A^{15} + \cos\theta B^3 \end{aligned} \tag{35}$$

where

$$\cos\theta = \frac{\sqrt{\frac{3}{2}} g_4}{c}, \qquad \sin\theta = \frac{g_R}{c} \quad : \quad c^2 = g_R^2 + \frac{3}{2} g_4^2 \tag{36}$$

It is convenient [1] to define the 4×4 (2×2) matrix $\mathbf{A}$ ($\mathbf{B}$) as follows

$$\mathbf{A} = \frac{T^b A^b}{\sqrt{2}} \text{ with } A^a_b \equiv (\mathbf{A})_{\mathbf{ab}} \quad , \quad \mathbf{B} = \frac{\tau^r B^r}{\sqrt{2}} \text{ with } B^r_s \equiv (\mathbf{B})_{\mathbf{rs}} \tag{37}$$

which leads to

$$A^4_4 = -\frac{\sqrt{3}}{2} A^{15} \quad , \quad B^1_1 = \frac{B^3}{\sqrt{2}} \tag{38}$$

In the notation of Eq. (37), the gaugino fields which lie in the same supermultiplet as the gauge fields A^a_b of the $SU(4)_C$ group are denoted by λ^a_b ($a, b = 1, \ldots, 4$ with $\lambda^a_a = 0$), whereas we denote the gaugino fields of the $SU(2)_{L,R}$ group by $\lambda^r_{s\,L,R}$ ($r, s = 1, 2$ with $\lambda^r_r = 0$). Then the gaugino mass term in the G_{422} group is:

$$\begin{aligned} \text{mass term} &= M_4 \lambda^a_b \lambda^b_a + M_L \lambda^r_{s\,L} \lambda^s_{r\,L} + M_R \lambda^r_{s\,R} \lambda^s_{r\,R} \\ &= M_4 \widehat{\lambda^\alpha_\beta} \widehat{\lambda^\beta_\alpha} + \frac{4}{3} M_4 (\lambda^4_4)^2 + M_L \lambda^r_{s\,L} \lambda^s_{r\,L} + 2 M_R (\lambda^1_{1\,R})^2 + \ldots \end{aligned} \tag{39}$$

where $\widehat{\lambda^\alpha_\beta} = \lambda^\alpha_\beta - \frac{1}{3}\delta^\alpha_\beta \lambda^\gamma_\gamma$ ($\alpha, \beta = 1, 2, 3$) are the $SU(3)_C$ gaugino fields and '...' denote the terms which do not contribute to the MSSM gaugino masses.

Since the gaugino mixing should proceed in the same way as that for the gauge fields lying in the same supermultiplet, then Eqs. (35 and 38) lead 'by supersymmetry' to:

$$\lambda^4_4 = -\frac{\sqrt{3}}{2}(\sin\theta\lambda + \cos\theta\widetilde{\lambda}) \tag{40}$$

$$\lambda^1_{1R} = \frac{1}{\sqrt{2}}(\cos\theta\lambda - \sin\theta\widetilde{\lambda}) \tag{41}$$

where λ is the gaugino field lying in the same supermultiplet as the $U(1)_Y$ gauge field E, whereas $\widetilde{\lambda}$ is the superpartner of the massive vector boson F.

It follows from Eq.(39) that at the intermediate scale M_{HB} we have:

$$M_3|_{M_{HB}} = M_4|_{M_{HB}} \quad , \quad M_2|_{M_{HB}} = M_L|_{M_{HB}} \tag{42}$$

As to the mass term corresponding to $U(1)_Y$, then substituting Eqs.(40 and 41) into Eq.(39) leads to:

$$M_1|_{M_{HB}} = \sin^2\theta M_4 + \cos^2\theta M_R = \left.\frac{2g_R^2 M_4 + 3g_4^2 M_R}{3g_4^2 + 2g_R^2}\right|_{M_{HB}} \tag{43}$$

To summarize, we have used an $SO(10)$–$\mathbf{16}$ irrep Higgs field to break G_{422} into the SM when its neutral component $(\mathbf{1},\mathbf{1})_0$ under SM develops a vev. The gauge supermultiplets $\mathbf{45}$ of $SO(10)$ would also be decomposed having under G_{422} the components $(\mathbf{15},\mathbf{1},\mathbf{1})$ and $(\mathbf{1},\mathbf{1},\mathbf{3})$representing respectively the generators of $SU(4)$ and $SU(2)_R$. In the breaking from G_{422} to SM, each of the latter generators would have a singlet $(\mathbf{1},\mathbf{1})_0$ part and one needs to identify the weak hypercharge Y generator as a linear combination of these $(\mathbf{1},\mathbf{1})_0$ parts. With this, we could determine the $U(1)_Y$ gaugino in terms of the gauginos and coupling constants g_4, g_R corresponding to $SU(4)_C$ and $SU(2)_R$.

3.2 $H \equiv SO(3) \times SO(7) \to H' \equiv SU(2)_L \times SO(6) = SU(2)_L \times SU(4) \to SM \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$

As we have discussed, one can use the irreps $\mathbf{54}$ or $\mathbf{770}$ to carry out the breaking $SO(10) \to SO(3) \times SO(7) \equiv SU(2)_L \times SO(7)$. As pointed out in [9], the $SU(2) \times SO(7)$ can not be reconciled with the chiral fermion content of the SM. However, as was noticed in [22], this case produces non-trivial mass ratios with interesting phenomenology, and we may still consider it since we are not involved in the model building. Thus, until the identification of a feasible model with masses in this region, we include the examination of this case in our study.

Now, the $SO(7)$ is broken at M_{GUT} to $SO(6) \simeq SU(4)$ which in turn is broken to $SU(3)_C \times U(1)_Y$ at M_{HB}. One can not use the $SU(4)$-$\mathbf{4}$ irrep to achieve this breaking since its branching rule is:

$$\mathbf{4} \overset{SU(4)\supset SU(3)\times U(1)}{=} \mathbf{1}_3 + \mathbf{3}_{-1} \tag{44}$$

whereas the 'next simple' $SU(4)$-$\mathbf{15}$ irrep can carry out this breaking having the branching rule:

$$\mathbf{15} \overset{SU(4)\supset SU(3)\times U(1)}{=} \mathbf{1}_0 + \mathbf{3}_{-4} + \overline{\mathbf{3}}_4 + \mathbf{8}_0 \tag{45}$$

Thus, the Higgs field Φ responsible for the breaking $SU(4) \to SU(3)_C \times U(1)_Y$ should include the $SU(4)$-$\mathbf{15}$ irrep, and the simplest choice is the $\mathbf{45}$ irrep of $SO(10)$ having the branching rules:

$$\mathbf{45} \overset{SO(10)\supset SO(3)\times S(7)}{=} (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{21}) + (\mathbf{3},\mathbf{7}) \tag{46}$$

$$\mathbf{21} \overset{SO(7)\supset SO(6)}{=} \mathbf{15} + \mathbf{6} \tag{47}$$

The $(SO(7))$ gaugino mass term in the Lagrangian is

$$\mathcal{L}^{SO(7)}_{\rm mass} \;=\; M_7\lambda^{[a,b]}\lambda_{[a,b]} = M_7\lambda^{[\alpha,\beta]}\lambda_{[\alpha,\beta]} + M_7\lambda^{[7,\alpha]}\lambda_{[7,\alpha]} \tag{48}$$

where $a,b = 1,\ldots,7; \alpha,\beta = 1,\ldots,6$. Note that the $\lambda^{[7,\alpha]}$ does not represent the superpartner of a gauge field in $SO(6) = SU(4)$, and thus, using the $SU(4)$ indices, the mass term of the $SU(4)\times SU(2)_L$ is

$$\begin{aligned}\mathcal{L}_{\rm mass} &= M_4'\lambda^i_j\lambda^j_i + M_L\lambda^r_s\lambda^s_r \\ &= M_4'\lambda^\alpha_\beta\lambda^\beta_\alpha + M_4'\lambda^4_4\lambda^4_4 + M_L\lambda^r_s\lambda^s_r + \ldots\end{aligned} \tag{49}$$

where $i,j = 1,\ldots,4$ (with $\lambda^i_i = 0$); $r,s = 1,2$ (with $\lambda^r_r = 0$); $\alpha,\beta = 1,2,3$ and the '$\ldots$' represent the terms which do not contribute to the gaugino masses: M_L for $SU(2)$ and M_4' for $SU(4)$ satisfying

$$\left.\frac{M_4'}{M_7}\right|_{M_{GUT}} = 1 \tag{50}$$

We introduce in the same way as we did before, the 'traceless' $SU(3)$-gauginos: $\widehat{\lambda^\alpha_\beta} = \lambda^\alpha_\beta - \frac{1}{3}\delta^\alpha_\beta\lambda^\gamma_\gamma$, and the 'squared' $U(1)_Y$ gaugino field $\lambda^2 = \frac{1}{3}(\lambda^\gamma_\gamma)^2 + (\lambda^4_4)^2$. Eq.(49) reduces then to

$$\mathcal{L}_{\rm mass} \;=\; M_4'\widehat{\lambda^\alpha_\beta}\widehat{\lambda^\beta_\alpha} + M_4'\lambda^2 + M_L\lambda^r_s\lambda^s_r \tag{51}$$

Therefore, we have at M_{HB}, the scale where the breaking of the intermediate group H' takes place, the relations:

$$M_1|_{M_{HB}} = M_3|_{M_{HB}} = M_4'|_{M_{HB}} \quad , \quad M_2|_{M_{HB}} = M_L|_{M_{HB}} \tag{52}$$

3.3 $H = H_{51} \equiv SU(5)\times U(1)_X \rightarrow SM \equiv SU(3)_C\times SU(2)_L\times U(1)_Y$

In order to break $SU(5)$ to $SU(3)_C\times SU(2)_L\times U(1)_Z$, one can use the (SU_5)-10-irrep with the branching rule:

$$\mathbf{10} \;\overset{SU(5)\supset SU(3)_C\times SU(2)_L\times U(1)_Z}{=}\; (\mathbf{3}^*,\mathbf{1})_{-\frac{2}{3}} + (\mathbf{3},\mathbf{2})_{\frac{1}{6}} + (\mathbf{1},\mathbf{1})_1 \tag{53}$$

Thus, the Higgs field Φ responsible for the breaking $H_{51}\rightarrow SM$ can be taken in the $(SO10)$-16-irrep having the branching rule:

$$\mathbf{16} \;\overset{SO(10)\supset SU(5)\times U(1)_X}{=}\; \mathbf{10}_1 + \bar{\mathbf{5}}_{-3} + \mathbf{1}_5 \tag{54}$$

The conventions in the above two branching rules are consistent with the $U(1)_Z$-generator in $SU(5)$ given by:

$$Z \;=\; diag(-1/3,-1/3,-1/3,1/2,1/2) \tag{55}$$

and we have an unbroken hypercharge [23]:

$$\frac{Y}{2} = \frac{1}{5}(X - Z) \tag{56}$$

As it is well known, one needs to define the 'properly normalized' $U(1)_Z$-generator to be:

$$L_Z \;=\; \sqrt{\frac{3}{5}}Z \tag{57}$$

so that $Tr(L_Z)^2 = \frac{1}{2}$. Similarly, we define the 'properly normalized' $U(1)_X$-generator to be:

$$L_X = \sqrt{\frac{1}{40}}X \tag{58}$$

such that $Tr_{\mathbf{10}}(L_X)^2 = 1$, since we should have $Tr_{\mathbf{10}}(M_{ij}M_{i'j'}) = 1\delta_{ii'}\delta_{jj'}$ where M_{ij} is the $SO(10)$ generator and **10** is the defining (vector) irrep of $SO(10)$, and that the branching rule

$$\mathbf{10} \overset{SO(10)\supset SU(5)\times U(1)_X}{=} (\mathbf{5})_2 + (\overline{\mathbf{5}})_{-2} \tag{59}$$

implies $Tr_{\mathbf{10}}(X^2) = 40$.

We now come to the mixing of the two $U(1)$'s, which means we study how $U(1)_Z \times U(1)_X$ breaks into $U(1)_Y$. When the Higgs field corresponding to the $(1,1)$ component of Eq. (53), with Z- and X-charges equal to one and represented by a 5×5 antisymmetric tensor ϕ^{ab}, takes a vev such that the only non-zero elements are:

$$<\phi^{45}> = -<\phi^{54}> = v \tag{60}$$

we get a mass term

$$\mathcal{L}_{\text{mass}} = v^2\left(g_5\sqrt{\frac{3}{5}}A^Z_\mu + \frac{g_X}{\sqrt{40}}B^X_\mu\right)^2 \tag{61}$$

where A^Z and B^X are the $U(1)_Z$ and $U(1)_X$ gauge fields, respectively.

By diaganolizing the mass matrix corresponding to the above quadratic form, we get a massive $U(1)_Y$-neutral vector boson field B_μ and a massless $U(1)_Y$-gauge field A_μ given by:

$$\begin{aligned} B &= \cos\psi A^Z - \sin\psi B^X \\ A &= \sin\psi A^Z + \cos\psi B^X \end{aligned} \tag{62}$$

where

$$\cos\psi = \frac{\sqrt{3}g_5}{c}, \qquad \sin\psi = -\frac{g_X}{\sqrt{8}c} \quad : \quad c^2 = 3g_5^2 + \frac{g_X^2}{8} \tag{63}$$

Let $\lambda, \widetilde{Z}$ be the superpartners of B^X, A^Z respectively, and call $\widetilde{X}$ the superpartner of the massive B, whereas we denote the superpartner of the massless A, that is the $U(1)_Y$ gaugino, by $\widetilde{Y}$. Then from Eq. (62) we have

$$\begin{aligned} \widetilde{X} &= \cos\psi\widetilde{Z} - \sin\psi\lambda \\ \widetilde{Y} &= \sin\psi\widetilde{Z} + \cos\psi\lambda \end{aligned} \tag{64}$$

The gaugino mass term of the $H_{51} \equiv SU(5)\times U(1)_X$ can be written as:

$$\mathcal{L} \supset M_5\lambda^a_b\lambda^b_a + M_X\lambda^2 \tag{65}$$

where λ^a_b's are the gauginos of $SU(5)$ ($a, b = 1,\ldots,5$ and $\lambda^a_a = 0$)†. After H_{51} is broken to the SM, with the indices ($\alpha, \beta = 1,2,3; r, s = 4,5$), we have:

$$\mathcal{L}_{\text{mass}} = M_5[(\widehat{\lambda^\alpha_\beta}\widehat{\lambda^\beta_\alpha})^2 + (\widehat{\lambda^r_s}\widehat{\lambda^s_r})^2 + \widetilde{Z}^2] + M_X\lambda^2 \tag{66}$$

†referred to by $\widehat{\lambda^a_b}$ in Eq. 16.

where $\widehat{\lambda_\beta^\alpha} = \lambda_\beta^\alpha - \frac{1}{3}\delta_\beta^\alpha \lambda_\gamma^\gamma$ are the gaugino fields of $SU(3)_C$ (Similarly, $\widehat{\lambda_s^r}$ are the $SU(2)_L$ gauginos) and $\widetilde{Z}^2 = \frac{1}{3}(\lambda_\alpha^\alpha)^2 + \frac{1}{2}(\lambda_r^r)^2$ is the squared $U(1)_Z$ gaugino field. From Eq.(66) and using Eq.(64) we get:

$$M_2|_{M_{HB}} = M_3|_{M_{HB}} = M_5|_{M_{HB}} \quad , \quad M_1|_{M_{HB}} = M_5 \sin^2\psi + M_X \cos^2\psi = \left.\frac{g_X^2 M_5 + 24 g_5^2 M_X}{g_X^2 + 24 g_5^2}\right|_{M_{HB}} \tag{67}$$

To summarize, we obtained by calculating the mixing of the two $U(1)$'s the formulae relating the MSSM-guagino masses (M_1, M_2, M_3) to the intermediate group H_{51}-gaugino masses (M_5, M_X) and the coupling constants, which are valid at the scale where the breaking of the intermediate group to the SM occurs.

4 The RG running and the MSSM gaugino mass ratios

In section 2, we computed the H-gaugino mass ratios at the GUT scale M_{GUT}, whereas in section 3 we expressed, at the intermediate breaking scale M_{HB}, the MSSM gaugino masses in terms of the H-gaugino masses and the coupling constants. Thus, it is necessary to introduce the running factors for the gauge couplings of the intermediate group ($\alpha_i \equiv \frac{g_i^2}{4\pi}$) from M_{GUT} to M_{HB}:

$$r_i = \frac{\alpha_i(t)}{\alpha_i(t_0)} \quad , \quad t = \log \frac{M_{GUT}^2}{Q^2} \tag{68}$$

with $Q^2 = M_{HB}^2$ and $t_0 = 0$ corresponding to $Q^2 = M_{GUT}^2$, and we assume unification at M_{GUT} ($\alpha_i(t_0) = \alpha$). We define the ratio

$$R(i,j) \equiv \frac{r_i}{r_j} = \frac{1 + \frac{\alpha}{2\pi} b_j t}{1 + \frac{\alpha}{2\pi} b_i t} \tag{69}$$

with b_i the beta function coefficients, and use the one-loop renormalization equations for the evolution of the gaugino masses and the coupling constants:

$$\frac{M_i(t)}{g_i^2(t)} = \frac{M_i(t_0)}{g_i^2(t_0)} \tag{70}$$

With this we can obtain our final results of the MSSM gaugino mass ratios at the intermediate scale M_{HB} as follows:

- <u>$SO(10) \to G_{422}$ by **54**</u>

  Eqs.(42,43) and (4)lead to

$$\frac{M_2(t)}{M_3(t)} = -\frac{3}{2} R(2_L, 4) \quad , \quad \frac{M_1(t)}{M_3(t)} = \frac{-5R(2_R,4)}{4R(2_R,4)+6} \tag{71}$$

  We note that we get the gaugino masses M_a(a=1,2,3) in the ratio $-\frac{1}{2} : -\frac{3}{2} : 1$ when the two scales are equal ($M_{HB} = M_{GUT}$) in accordance with the results of [9] obtained via a different approach. However, it is instructive to notice here that the functional form of the ratio M_1/M_3, in terms of the 'RG'-factor $R(2_R, 4)$, in equation (71) can not be deduced directly, by simple RG running, from its value ($-\frac{1}{2}$) when $R(2_R, 4) = 1$ corresponding to two equal scales. This comes because the mixing of two $U(1)$'s, one from $SU(4)_C$ and the other from $SU(2)_R$, to give $U(1)_Y$ happens at the intermediate scale M_{HB}, and use of Eq.(43) is essential in order to take account of this mixing.

- <u>$SO(10) \to G_{422}$ by **210**</u>

  Eqs.(42,43) and (8,9) lead to

$$M_3(t) = 0 \quad , \quad \frac{M_1(t)}{M_2(t)} = \frac{-3}{3 + 2R(2_R,4)} \tag{72}$$

where the symmetric evolution of α_{2R} and α_{2L} puts $R(2_R, 2_L) = 1$. This reduces to the 'known' value $\frac{M_1}{M_2} = -\frac{3}{5}$ when $M_{HB} = M_{GUT}$ [9]. We note that the possibility of gluinos being massless is not phenomenologically excluded.

- $\underline{SO(10) \to G_{422} \text{ by } \mathbf{770}}$

  Eqs.(42,43) and (23) lead to

$$\frac{M_1(t)}{M_3(t)} = \frac{19R(2_R, 4)}{6 + 4R(2_R, 4)} \quad , \quad \frac{M_2(t)}{M_3(t)} = \frac{5}{2}R(2_L, 4) \tag{73}$$

  We see that when $M_{HB} = M_{GUT}$ the results of the gaugino masses M_a(a=3,2,1) reduce, as expected, to $1 : \frac{5}{2} : \frac{19}{10}$ in ratio [9].

- $\underline{SO(10) \to SU(2) \times SO(7) \text{ by } \mathbf{54}}$

  Eqs. (50,52) and (6) lead to gaugino masses, at the intermediate scale M_{HB}, in the ratio:

$$M_3 : M_2 : M_1 \quad = \quad 1 : -\frac{7}{3}R(2_L, 4) : 1 \tag{74}$$

  which reduces to $1 : -\frac{7}{3} : 1$ when $M_{HB} = M_{GUT}$ [7].

- $\underline{SO(10) \to SU(2) \times SO(7) \text{ by } \mathbf{770}}$

  Eqs. (50,52) and (25) lead to

$$\frac{M_1(t)}{M_3(t)} = 1 \quad , \quad \frac{M_2(t)}{M_3(t)} = 7R(2_L, 4) \tag{75}$$

  which reduce respectively to $1, 7$, when $M_{HB} = M_{GUT}$.

- $\underline{SO(10) \to H_{51} \text{ by } \mathbf{210}}$

  Eqs. (67) and (18) lead to

$$\frac{M_2(t)}{M_3(t)} = 1 \quad , \quad \frac{M_1(t)}{M_3(t)} = \frac{-95R(1_X, 5)}{R(1_X, 5) + 24} \tag{76}$$

  Again, these functional forms are consistent with the 'known' values of the gaugino mass M_a(a=3,2,1) ratios $1 : 1 : -\frac{19}{5}$ obtained in [9] using a different method when $M_{HB} = M_{GUT}$. However, their values at M_{GUT} and RG running alone are not enough to deduce the 'functional' forms, and one needs to carefully consider the normalization and mixing of $U(1)_X$ and $U(1)_Z$, which was done in Eq.(67)

- $\underline{SO(10) \to H_{51} \text{ by } \mathbf{770}}$

  Eqs. (67) and (29) lead to

$$\frac{M_2(t)}{M_3(t)} = 1 \quad , \quad \frac{M_1(t)}{M_3(t)} = \frac{385R(1_X, 5)}{24 + R(1_X, 5)} \tag{77}$$

  which reduce respectively to $1, \frac{77}{5}$ if $M_{HB} = M_{GUT}$, in accordance with [9].

We compute now the beta coefficients for the RG running. We shall consider that the scale M_{HB} is above the threshold of creating the superpartners of the known particles, so we use the RG equations of the SUSY-GUT [24]:

$$b_i \quad = \quad S_i(R) - 3C_i(G) \tag{78}$$

with $S_i(R)$ is the Dynkin index of the irrep R summed over all chiral superfields, normalized to 1/2 for each fundamental irrep of $SU(N)$, and $C_i(G)$ is the Casimir invariant (equal to the Dynkin index of the adjoint representation) which satisfies $C(SU(N)) = N, C(U(1)) = 0$. In order to single out the Higgs contribution, we write:

$$S_i(R) \quad = \quad F_i + H_i \tag{79}$$

and we shall assume we have $N_g = 3$ families of fermions which span an $SO(10)$-**16** spinor irrep.

As to the Higgs field, we only consider the Higgs field responsible for the breaking of the intermediate group H. These Higgs fields would include the MSSM Higgses but the way in which this is carried out is model-dependent. As to the Higgs fields responsible for the breaking of $SO(10)$, we do not consider them since they get masses of order of M_{GUT}, and some will be 'eaten' by the gauge bosons.

As explained in section 3, we need a Higgs field Φ in an **16**-irrep of $SO(10)$ in both cases corresponding to $H = G_{422}$ and $H = H_{51}$, whereas we need a Higgs field Φ in an **45**-irrep of $SO(10)$ in the case $H = SU(2) \times SO(7)$, whence we have the table:

H	Higgs	F_i	H_i	C_i	b_i	F_j	H_j	C_j	b_j	MSSM
G_{422}	**16**	2	2	2	2	2	2	4	-4	$b_1^{MSSM} = \frac{33}{5}$
$SU2 \times SO7$	**45**	4	16	2	22	2	8	4	2	$b_2^{MSSM} = 1$
H_{51}	**16**	2	2	0	8	2	2	5	-7	$b_3^{MSSM} = -3$

Table 1: $(i, j) = (2_R, 4)$ or $(2_L, 4)$ for $H = G_{422}$ or $H = SU2{\times}SO7$ (broken at M_{GUT} to $H' = SU2{\times}SU4$), whereas $(i, j) = (1_X, 5)$ for $H = H_{51}$. We put also the MSSM beta function coefficients.

Irrep	H	M_1/M_3				M_2/M_3			
$M_{HB} =$			M_{GUT}	10^8	10^3		M_{GUT}	10^8	10^3
54	G_{422}	$\frac{-5R(2_R,4)}{6+4R(2_R,4)}$	$-1/2$	0.88 (3.21)	2.27 (1.96)	$-\frac{3}{2}R(2_L,4)$	$-3/2$	0.93 (3.13)	1.45 (1.58)
	$SU2 \times SO7$	1	1	1 (-6.42)	1 (-3.92)	$-\frac{7}{3}R(2_L,4)$	$-7/3$	-0.36 (4.88)	-0.31 (2.45)
210	G_{422}	$m = \frac{-3}{3+2R(2_R,4)}$	$m = -\frac{3}{5}$	$m = -1.70$ ($= -1.84$)	$m = -2.82$ ($= -2.24$)	∞	∞	∞	∞
	H_{51}	$\frac{-95R(1_X,5)}{24+R(1_X,5)}$	$-19/5$	2.21 (24.38)	2.67 (14.90)	1	1	1 (-2.09)	1 (-1.05)
770	G_{422}	$\frac{19R(2_R,4)}{6+4R(2_R,4)}$	19/10	-3.34 (-12.19)	-8.63 (-7.45)	$\frac{5}{2}R(2_L,4)$	5/2	-1.55 (-5.22)	-2.42 (-2.63)
	$SU2 \times SO7$	1	1	1 (-6.42)	1 (-3.92)	$7R(2_L,4)$	7	1.09 (-14.63)	0.93 (-7.36)
	H_{51}	$\frac{385R(1_X,5)}{24+R(1_X,5)}$	77/5	-8.95 (-98.80)	-10.85 (-60.40)	1	1	1 (-2.09)	1 (-1.05)

Table 2: Gaugino mass ratios at intermediate scale M_{HB} in the different cases. To each ratio correspond four columns, the first of which gives the general formula whereas the other three give the result when M_{HB} is taking a specific value. Bracketed values denote the gaugino mass ratios when $M_{HB} = M_{GUT}$ evaluated at the same specific energy scale (10^3 or 10^8 GeV) as the case of $M_{HB} \neq M_{GUT}$. The following numerical values are taken: $M_{GUT} = 10^{16}$, $\alpha = 0.1$. Mass scales are evaluated in GeV. The parameter m is equal to $\frac{M_1}{M_2}$.

5 Summary and Discussion

We summarize our results in Table 2, where we compute the gaugino mass ratios in the different cases, using equation (69), with $\alpha \sim 0.1$, $M_{GUT} = 10^{16}$ GeV and we take two values for the intermediate breaking scale $M_{HB} = 10^3, 10^8$ GeV.

In order to illustrate in the table the effect of 'successive' breakings, we have enclosed in brackets the values of the gaugino mass ratios at the specific values $10^3, 10^8$ GeV, had the two breakings occurred at one stage ($M_{HB} = M_{GUT}$), using the MSSM running from $E = M_{GUT}$ to $E = 10^3$ or 10^8 GeV:

$$\frac{M_i}{M_j}(E) \quad = \quad \frac{M_i}{M_j}(M_{GUT})\frac{1+\frac{\alpha}{2\pi}tb_i^{MSSM}}{1+\frac{\alpha}{2\pi}tb_j^{MSSM}} \tag{80}$$

where $t = \log(\frac{M_{GUT}}{E})^2$.

We see that gaugino mass ratios, evaluated at the same energy scale, change significantly when the intermediate scale is low (say, 10^8 GeV or Tev) compared to when the two breaking scales are approximately equal.

We note here that we did not consider the impact of the intermediate scale on gauge coupling unification for the values of the parameters used in the table. To check that this unification requirement can be achieved in a way consistent with the low scale experimental measurements would involve model building details, where one constructs a complete SUSY GUT model with a full superpotential explicitly written, and in which the gauge coupling unification is realized in two steps of breaking: a task beyond the scope of the work in this paper which does not entail model building particularities.

Having said this though, one should notice that from a phenomenological point of view there is a more reasonable way to obtain the gaugino mass ratios at the intermediate scale M_{HB}. In fact, once we fix the partially unified intermediate gauge group H and the intermediate mass scale M_{HB}, the values of the gauge couplings at M_{HB} can be calculated from the weak scale data by using RG equations, and then one can use the formulae of the past section to compute the corresponding gaugino mass ratios assuming gauge coupling unification at M_{GUT}. However, whether or not the numerical values of the running gauge couplings at a 'low' intermediate scale M_{HB} ‡, which are necessary to evaluate the gaugino mass ratios at this scale, can match with the SM gauge couplings measured at the electroweak scale M_Z, provided we insist on having just MSSM between M_{HB} and M_Z §, would depend heavily on the nature of H. For instance, if $H = SU(5) \times U(1)$, it is difficult to get a low intermediate mass scale and unify both coupling constants to one corresponding to $SO(10)$ [25]. Nonetheless, if $H = G_{3221} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the low intermediate mass scale can be obtained [16].

As an illustrative example, let us take the case of $H = G_{422}$ and calculate the gaugino mass ratios by way of computing the values of the gauge couplings at M_{HB} from the weak scale data, and assuming gauge coupling unification at M_{GUT} (which can be realized by, say, adding some particle content near M_{HB} similar to that in [16]¶). With the numerical values [26] ($M_Z = 91.18$ GeV, $\alpha_S(M_Z) \sim 0.1187$, $\sin^2\theta_W \sim 0.2312, \alpha_{em}^{-1}(M_Z) = 127.9 \Rightarrow \alpha_{2L}^{-1}(M_Z) = 29.57$ and $\alpha_Y^{-1}(M_Z) = 58.99$) and the MSSM beta coefficients from Table 1, we get, for $M_{HB} = 10^4 GeV$, the values: $\alpha_S^{-1} = 12.91, \alpha_{2L}^{-1} = 28.07, \alpha_Y^{-1} = 49.12$ at M_{HB}. Because H breaks into the SM at M_{HB}, we have $g_4(M_{HB}) = g_S(M_{HB})$ and $g_{2R}(M_{HB}) = g_{2L}(M_{HB})$. Applying Eqs. 71, 72 and 73, we get the numerical results of the gaugino mass ratios and show them in Table 3.

In general, considering other models and other intermediate groups, one can say that although some model complexifications might affect the coupling constants evolution, and consequently the values of the

‡By 'low' we mean a scale smaller than $\sim 10^{12} GeV$, so that to be capable of explaining the smallness of neutrino masses.

§More precisely, one has MSSM between M_{HB} and M_S, the SUSY scale, and SM between M_S and M_Z.

¶In [16], with the intermediate group G_{3221} and additional light supermultiplets with masses around the intermediate mass scale M_R (corresponding to M_{HB} in the present paper), one could, within SUSY SO(10) GUT, achieve low values for M_R ($10^4 - 10^{10} GeV$) with $M_{GUT} \sim 10^{16} GeV$.

Irrep	M_1/M_3	M_2/M_3
54	-0.29	-0.68
210	∞	$M_1/M_2 = -0.76$
770	1.11	1.15

Table 3: Gaugino mass ratios at $M_{HB} \sim 10$ TeV for an intermediate G_{422} group, obtained by computing the values of gauge couplings at M_{HB} starting from the weak scale data.

derived gaugino mass ratios, however the conclusion concerning the significant influence of the existence of multi-stages in the breaking chain would remain unchanged. The derived mass ratios would be reflected in the electroweak energy scale measurements due to take place in the near future experiments, like the LHC, with interesting phenomenological consequences.

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